WESTMINSTER SCHOOL
THE CHALLENGE 2016

MATHEMATICS III

Wednesday 27 April 2016

Time allowed: 1 hour 30 minutes

You may not use a calculator for this paper.

All your working should be clearly shown.

You should attempt all the questions.

Please write in black or blue ink.

1 To leave the jungle, George swims for 20 minutes at 1·8 kilometres per hour and runs 9 kilometres at 5 metres per second. What is George's average speed in metres per second over this journey?

2 One third of a cake weighing one and a quarter kilograms consists of chocolate. Sam ate all of this cake. Two fifths of a cake weighing two kilograms consists of chocolate. Sam ate some of this second cake. Sam ate 950 grams of chocolate altogether. What fraction of the second cake did he eat?

3 **a** Find the lowest common multiple of 34 and 35.

b Two sequences of numbers are given.
Sequence **A** starts at 20 and goes up in steps of 35.
Sequence **B** starts at 14 and goes up in steps of 34.

i Show clearly that 1000 appears in both sequences.
ii Explain how you can tell that 1000 is the smallest number which appears in both sequences.
iii Find the next number after 1000 that appears in both sequences.
iv What is the next number after 1000 that appears in both sequences and is also a multiple of 1000?

4 Alfie, Ben and Charlie are brothers. In a 100m race, Alfie beats Ben by 15 metres. In a second 100 m race, Ben beats Charlie by 20 metres. Assuming each boy runs each of his races at the same constant speed, by how many metres would Alfie beat Charlie in a 100m race?
[In a race, X beats Y by *n* metres if Y is *n* metres behind when X crosses the finishing line.]

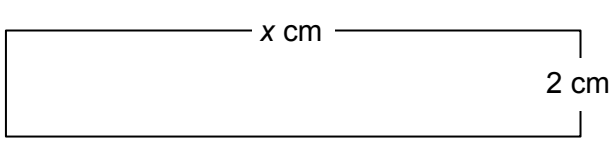
5 The sheep in two fields need to be sheared. There are 50% more sheep in the large field than in the small field. In the morning, a team of shearers, who all work at the same rate, spend 4 hours in the large field. In the afternoon, one third of the team spend another 4 hours in the large field, and the remaining two-thirds of the team spend four hours in the small field. After this, all the sheep in the large field have been shorn, and the next day it takes one man 8 hours to finish shearing the unshorn sheep in the small field.

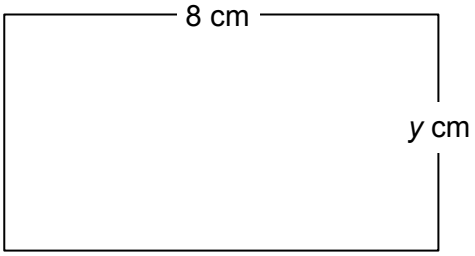
How many men are in the team of shearers?

6 Tom has three bags of apples. Prove that the following statements *cannot* all be true.

- There are different numbers of apples in each of the three bags;
- the largest bag contains six more apples than the smallest bag;
- the mean number of apples in the three bags added to the median number of apples in the three bags is equal to 23.

7 The diagram shows two rectangles: one has length x cm and width 2 cm, the other has length 8 cm and width y cm. You are given that $x > 8$, and $8 > y > 2$.



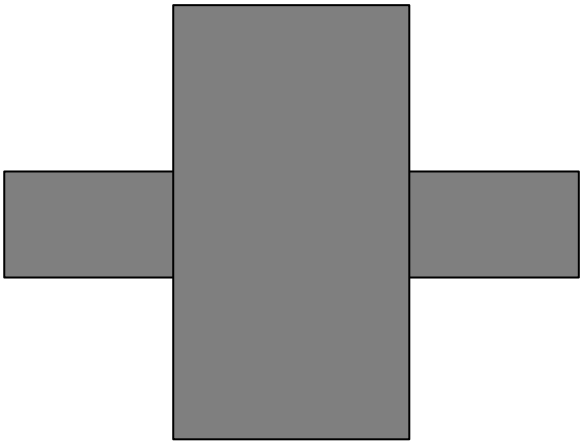


When the two rectangles are placed like this



the area they cover (shaded) is 49 cm^2.

When the two rectangles are placed like this



the area they cover (shaded) is 54 cm^2.

What are the values of x and y?

8 **a** Standard antifreeze mixture consists of 30% glycol and 70% water (by volume).

i What volume of *water* would have to be added to 6 litres of this standard mixture to obtain a more dilute mixture which consisted of 20% glycol and 80% water?

ii What volume of *glycol* would have to be added to 6 litres of this standard mixture to obtain a more concentrated mixture which consisted of 40% glycol and 60% water?

b Ben has a bucket containing 6 litres of standard antifreeze mixture, and a large container of very concentrated antifreeze, which consists of 60% glycol and 40% water.

How much of the very concentrated antifreeze mixture should he add to his bucket so that the resulting mixture consists of 42% glycol and 58% water?

9 Andrew buys some fudge in a shop. It costs him £90.

Ben and Charlie both spend the same amount as each other on fudge, but they do not spend as much as Andrew.

- Ben buys 5 kilograms less fudge than Andrew, but at the same price per kilogram.
- Charlie buys the same number of kilograms of fudge as Andrew, but at a different shop where he pays £2 per kilogram less.

How many kilograms of fudge did Andrew buy?

10 **a** **i** Simplify $5(4a + 5b - 3c) - 3(2a + 7b - c)$

ii Given that

$4a + 5b - 3c = 28$

and

$2a + 7b - c = 30$,

find the value of

$7a + 2b - 6c$.

b **i** What number could be under the rectangular patch to make the simplification correct?

$$3(5a - 7b + 2c) - \square(a + 2b - c) = 11a - 29b + 10c$$

ii Show that there is no number which could be under the rectangular patch to make this simplification correct.

$$\square(3a + 2b + c) - 4(2a - b - 3c) = 13a + 18b - 5c$$

iii The simplification below is correct. What two numbers are under the two rectangular patches?

$$\square(9a - 7b - c) - \square(3a - 3b - 2c) = 3a + b + 8c$$

c Given that

$4a + 5b + 7c = 76$

and

$3a + 6b + 11c = 112$,

find the value of

$5a + 4b + 3c$.

11 Consider the following argument:

$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10$

$= 1 + 2 + 3 + 4 + 5 + (1 + 5) + (2 + 5) + (3 + 5) + (4 + 5) + (5 + 5)$

$= (1 + 2 + 3 + 4 + 5) + (1 + 2 + 3 + 4 + 5) + (5 + 5 + 5 + 5 + 5)$

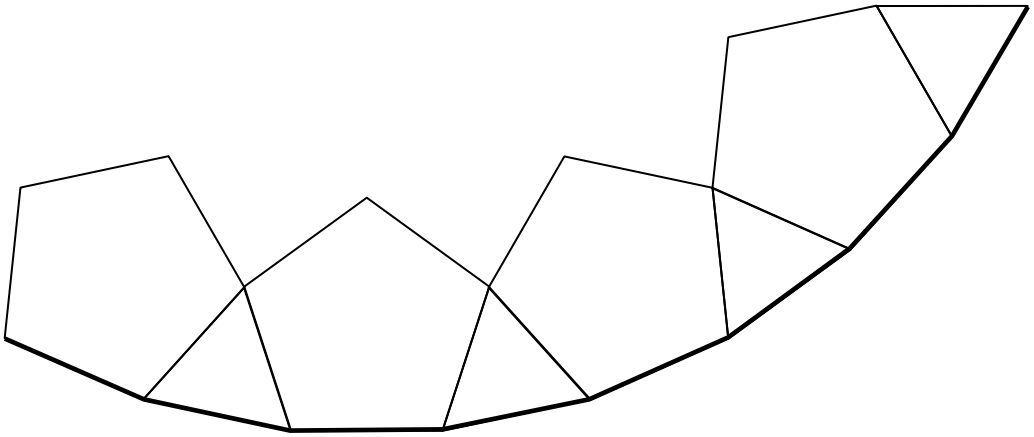
$= 2 \times (1 + 2 + 3 + 4 + 5) + 5^2$

$= (2 + 4 + 6 + 8 + 10) + 5^2$.

Write a generalisation of this argument to show that the sum of all the odd numbers between 1 and $2N$ is always N^2.

TURN OVER

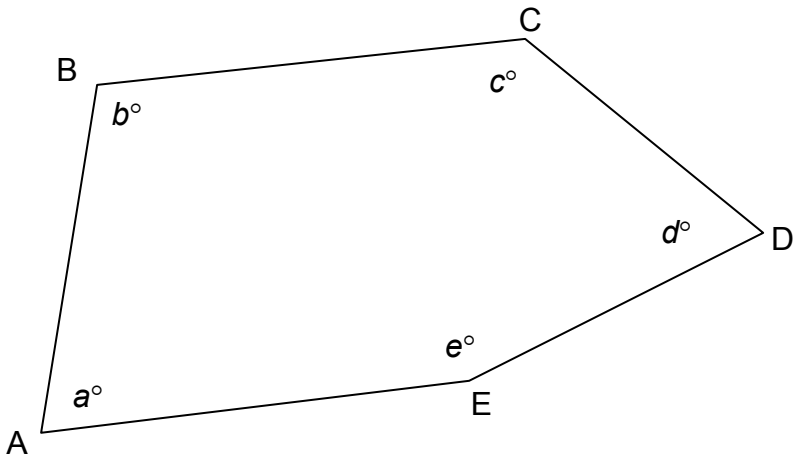
12 **a** The diagram below shows alternating equilateral triangles and regular pentagons. If the pattern is continued, the heavy line shown will eventually form a complete regular polygon. How many sides will this regular polygon have?



b The diagram shows a pentagon which is *not* regular. Prove that if

$c + d + e = 2(a + b)$

then BC is parallel to AE.



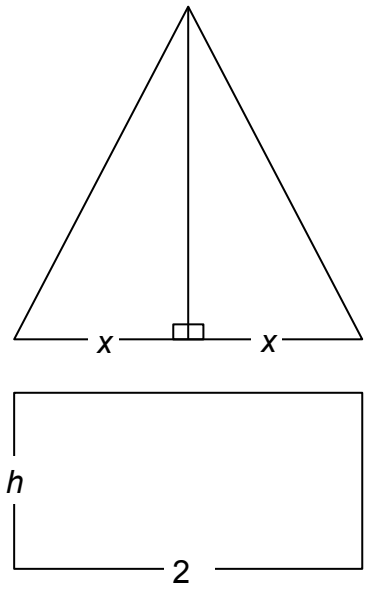
13 A hockey coach wishes to reduce the size of her squad, which includes boys and girls, by cutting five players. She calculates that:

- if all five players cut were boys, there would be equal numbers of boys and girls in the squad;
- if all five players cut were girls, there would be twice as many boys as girls in the squad.

If she wants the fraction of boys in the reduced squad to be $\frac{3}{5}$, how many boys and how many girls should she cut?

14 **a** Multiply out $(x+h)^2$

b The diagram shows an isosceles triangle and a rectangle. The triangle and the rectangle both have the same width: 2*x* centimetres. The height of the rectangle is *h* centimetres.



The triangle and the rectangle have the same area.

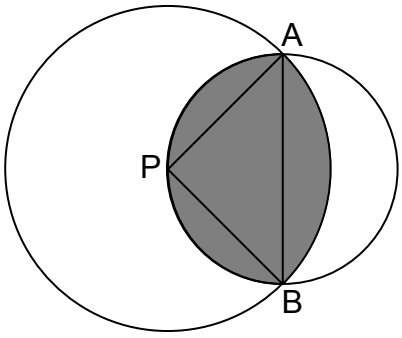
i Find the height of the triangle in terms of *h*.

ii Write an expression, in terms of *h* and *x*, for the perimeter of the triangle.

The triangle and the rectangle have the same perimeter.

iii Find *x* in terms of *h*.

15 In the diagram below, APB is a right angle, the larger circle has centre P, and the centre of the smaller circle is the midpoint of AB.



Given that the shaded area is equal to $18(\pi - 1)$ cm^2, find the radius of the larger circle.